Teaching Parallelograms Using Geometer's Sketchpad

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This study aims to examine the extent of the impact of using the Geometer's Sketchpad program (GSP) on learning the concept of parallelograms among fourth-grade students in the Arab sector in Israel and its influence on their performance in geometry, from the perspective of teachers. Additionally, the study seeks to explore the effect of variables such as teachers' educational qualifications and years of teaching experience.

To achieve these objectives, the researcher employed a descriptive-analytical methodology, utilizing a questionnaire composed of 20 items. The questionnaire was distributed to a sample of 100 fourth-grade teachers from various regions within the Arab sector. Following the distribution process, the responses were collected, coded, and inputted into a computer for statistical analysis using the Statistical Package for the Social Sciences (SPSS).

The results of the study indicated a very high degree of positive responses to the primary research question regarding the impact of the (GSP) on teaching parallelograms to fourth-grade students and its effect on their academic performance in geometry, as perceived by teachers. Furthermore, no statistically significant differences were found based on variables such as educational qualifications or years of teaching experience.

In light of these findings, the researcher recommends enhancing the utilization of the (GSP) in teaching mathematics and increasing the level of teacher training in this area. Additionally, the study emphasizes the importance of conducting broader investigations on this topic to further explore its potential.

Keywords: Geometer's Sketchpad, Dynamic Geometry, Parallelograms Education, Fourth-Grade Students, Mathematics Technology Integration.

INTRODUCTION

In the contemporary technological era, characterized by rapid advancements across all aspects of life, education is increasingly influenced by societal and personal demands. The accelerating pace of technological transformations compels educators to continuously seek innovative methods to improve educational programs, aligning them with the evolving demands of the modern world (Alhashmi et al., 2021). This has led to a shift in focus toward creating educational environments that not only capture students' interest but also foster the exchange of ideas and experiences. Such an approach ensures that learners across all levels can leverage contemporary technological tools to acquire knowledge and skills relevant to their era (Cirneanu & Moldoveanu, 2024).

Mathematics, as a core academic discipline, holds a prominent position in developing cognitive abilities due to its broad scope of knowledge. Its direct and indirect applications in everyday life underscore its importance within education systems globally (Sitopu et al., 2024). Beyond mere computational skills, modern educational goals emphasize understanding, critical thinking, and problem-solving, which align with evolving pedagogical methodologies (Engelbrecht & Borba, 2024). As technology continues to redefine learning paradigms, tools like Geometer's Sketchpad (GSP) exemplify how mathematics education can be enhanced to foster deeper conceptual understanding and engagement (Iji et al., 2020).

Geometry, a fundamental branch of mathematics, bridges theoretical concepts with real-world applications, including architectural design, mechanical systems, and technological devices (Çekmez, 2020). Its role in enhancing spatial reasoning and visual thinking makes it indispensable across various technological domains such as civil and mechanical engineering (Ahsan & Cahyono, 2020). Furthermore, the integration of geometry into modern technological frameworks enables students to develop skills essential for analyzing and understanding complex structures (Hibi, 2024a).

The intersection of mathematics and geometry emphasizes their mutual reinforcement. Geometry serves as a bridge between mathematical theory and practical applications, positioning it as a cornerstone of scientific and technological advancement (Hibi, 2024b). Integrating technology into this domain, through tools like GSP, transforms learning by offering students opportunities to interact with dynamic representations of abstract concepts (Alqahtani et al., 2022).

Interactive software and e-learning platforms have revolutionized mathematics education, making it more accessible and engaging. Numerous online platforms offer simplified and interactive content, enabling learners to grasp complex concepts effectively. These technologies ensure that mathematics education is available globally, bridging gaps in accessibility and fostering equity among diverse learner populations (Fokuo et al., 2023).

Geometer's Sketchpad (GSP) stands out as a versatile tool in this transformative landscape. Its applications span algebra, geometry, and analytical computations, making it a comprehensive resource for teaching and learning. By allowing users to draw geometric shapes, perform algebraic operations, and analyze mathematical relationships, GSP empowers educators and learners alike (Sebsibe et al., 2012). Available in various formats—desktop applications, online tools, and mobile apps—GSP provides an interactive platform for exploring and understanding mathematical and geometric concepts more effectively (Mazana et al, 2023).

In conclusion, the integration of technological tools like Geometer's Sketchpad enriches mathematics and geometry education, offering students a dynamic and interactive learning experience. These advancements not only enhance conceptual understanding but also prepare learners for real-world applications, bridging the gap between theoretical knowledge and practical skills. As educators continue to harness such tools, the potential for transformative learning experiences becomes increasingly evident.

Based on this foundation, this study seeks to investigate fourth-grade students' understanding of parallelograms—a fundamental geometric shape—using (GSP). The study also examines the software's impact on developing self-directed learning skills and academic achievement.

(GSP) is a versatile software tool that can be utilized for algebra, geometry, and analytical calculations. It enables students to draw geometric shapes using coordinates or points, supports the Arabic language, and is designed to help students independently acquire mathematical concepts (Semenikhina et al., 2020). The software provides comprehensive resources to make the learning process enjoyable and engaging, allowing students to build on prior knowledge (Zonzini et al., 2016).

Study Objectives and Significance: This study examines the impact of (GSP) on teaching parallelograms to fourth-grade students and its effect on their performance in geometry, addressing the following main question: What is the impact of using (GSP) on learning parallelograms and improving fourth-grade students' academic performance in geometry from teachers' perspectives?

From this primary question, the following sub-questions are derived:

1. What is the impact of (GSP) on fourth-grade students' understanding of parallelograms?

2. How does (GSP) influence fourth-grade students' academic performance in geometry?

Research Hypotheses:

1. There are no statistically significant differences at the ($\propto \leq 0.05$) level regarding the effect of (GSP) on fourth-grade students' academic performance in geometry from teachers' perspectives, based on differences in teachers' educational qualifications.

2. There are no statistically significant differences at the ($\propto \leq 0.05$) level regarding the effect of (GSP) on fourth-grade students' academic performance in geometry from teachers' perspectives, based on differences in years of teaching experience.

3. There are no statistically significant differences at the ($\propto \leq 0.05$) level regarding the effect of (GSP) on fourth-grade students' understanding of parallelograms, based on differences in teachers' educational qualifications.

4. There are no statistically significant differences at the ($\propto \leq 0.05$) level regarding the effect of (GSP) on fourth-grade students' understanding of parallelograms, based on differences in years of teaching experience.

Theoretical Significance: This research provides a conceptual framework that explores one of the contemporary teaching strategies: instruction through the use of the (GSP). It delves into how this tool can be utilized to teach fourth-grade students about quadrilaterals, with a particular focus on parallelograms. The study emphasizes the processes of constructing and understanding the properties of parallelograms, contributing to the theoretical foundation of integrating technological tools in mathematics education.

Practical Significance: The study highlights the importance of employing educational software, such as (GSP), in teaching the proposed unit. It also demonstrates how the tool can facilitate self-directed learning among students by engaging them with interactive, technology-based methods. Additionally, the research aims to equip primary school mathematics teachers with innovative instructional approaches, thereby advancing the teaching of geometry through modern, effective strategies.

The primary aim of this study is to examine the impact of using (GSP) on teaching the concept of parallelograms to fourth-grade students and its influence on their academic achievement in geometry, as perceived by their teachers.

This translation reflects an academic tone suitable for a high-level mathematics education journal, emphasizing clarity and precision. Let me know if you'd like further adjustments.

THEORETICAL FRAMEWORK

Teaching Mathematics and Geometry

Mathematics stands as one of the disciplines most influenced by the technological revolution. The evolution of tools such as computers, smartphones, accounting software, and engineering applications has not only enriched mathematical concepts but also highlighted the dynamic and transformative nature of this science. These advancements have significantly propelled the role of mathematics in the marathon of knowledge and creative innovation (Bruhn & Lüken, 2023).

As a multi-faceted field, mathematics plays a pivotal role in fostering cognitive development due to its direct and indirect applications in daily life. This centrality has positioned mathematics prominently within educational curricula.

Moreover, the emphasis in modern education has shifted from mere acquisition of mathematical knowledge and computational skills—now efficiently handled by calculators and computers—to a focus on comprehension, critical thinking, and problem-solving abilities. These objectives align with contemporary pedagogical strategies aimed at cultivating deeper intellectual engagement (Van & Drijvers, 2020).

Mathematics is inherently abstract, a construct of human intellect. Its evolution as a universal language, utilizing precise symbols and expressions, facilitates intellectual communication and fosters meaningful interaction with reality. Recognized as an independent and integrated system of knowledge, mathematics is crucial for organizing ideas and understanding the environment. Its applications extend across diverse fields, reinforcing its unique standing among the sciences. Consequently, modern mathematics curricula have evolved to address contemporary societal needs, extending beyond content to encompass teaching methodologies and tools that deliver mathematical knowledge with ease and clarity (Hibi, 2024c).

Geometry, as an integral branch of mathematics, further enriches its educational value. It equips learners with essential skills for practical life, such as spatial reasoning, exploration, problem-solving, deductive reasoning, and hypothesis formulation. Geometry also bridges cognitive learning domains by linking various mathematical branches. By employing modern educational technologies and teaching tools, such as computers, interactive software, the internet, multimedia technologies, and e-learning platforms, educators can effectively equip students with life skills rather than focusing solely on delivering factual knowledge (Szabo et al, 2020). These tools not only make mathematical and geometric concepts more accessible but also foster exploration, critical thinking, and creativity, making them indispensable in modern education.

In conclusion, the integration of mathematics and geometry in the educational landscape underscores their fundamental role in shaping intellectual capabilities. Modern technologies serve as transformative tools that redefine teaching strategies, enhancing both engagement and understanding in the classroom.

The Academic Achievement of Arab Students in Israel

Israel is home to approximately 2 million Arab citizens, geographically distributed across three main regions: The Galilee, the Mosholash, and the Negev. Schools in the Arab sector operate under similar resource allocations as those in the Jewish sector, adhering to the national curriculum. By the end of sixth grade, students in these schools are expected to have mastered basic computational skills, including the four arithmetic operations and operations involving simple and decimal fractions, taught through standard written algorithms within a traditional instructional framework. In junior high, students are introduced to more complex algebraic concepts while continuing to develop computational proficiency.

The results of the 2022 PISA tests, published in December 2023, reveal that Israeli students rank 38th in mathematics and 37th in science among the participating countries. These rankings underscore the significant underperformance of Israeli students compared to the international average (OECD, 2023a). A deeper analysis highlights that the mathematics and science scores of Israeli students fall well below the average achieved by 625,000 students across 81 participating countries (OECD, 2023b). Furthermore, a report by the National Authority for Measurement and Evaluation in Education in Israel (2023) indicates that over 60% of Arab-Israeli students tested were identified as struggling across all content areas evaluated.

The achievement gap in mathematics between Jewish and Arab students in Israel is particularly pronounced. According to the National Authority for Measurement and Evaluation in Education in Israel (2023), 69% of Arab students scored at levels 1-2 (out of six levels), compared to 31% of students in Hebrew-speaking schools. The gap between the performance of Arab and Jewish students is wider than the disparity between Israel and the average performance of OECD countries.

Tzuriel et al. (2023) identify several challenges within the Arab education sector, including reliance on outdated teaching methods, a focus on assessment rather than pedagogy, and low teacher salaries and status, which limit the recruitment of high-quality educators. Additionally, the per-student budget allocated to Arab education remains below international standards. Socio-economic disparities, as highlighted by David (2023) and Sabra and Alshwaikh (2023), further contribute to the achievement gaps between Arab and Jewish students. Despite these obstacles, efforts are being made to close these gaps by improving resource allocation and adopting better educational practices to enhance the quality of education for all students.

One significant factor influencing the underperformance of Arab students in international mathematics assessments is the heavy reliance on algorithmic practices in their mathematical instruction. This dependence has become ingrained in the mathematical identity of Arab students, fostering familiarity with procedural problem-solving without cultivating deep conceptual understanding or advanced mathematical thinking. Sharkia and Kohen (2021), in their study titled "Class Flipping Among Minorities in the Context of Learning Mathematics: The Israeli Case," investigated high-achieving Arab high school students. They observed that algorithmic techniques were prevalent, yet often lacked the necessary depth to foster a robust mathematical foundation.

The findings of these studies strongly support the need for integrating innovative numerical methods into the mathematics curricula of Arab schools in Israel. This integration seeks to address existing educational disparities, strengthen students' mathematical competencies, and promote analytical reasoning. By doing so, the educational framework in the Arab sector could move closer to achieving equity and excellence. Our research aims to further substantiate this argument and propose actionable strategies to enhance mathematical education within the Arab sector.

E-Learning: A Transformative Tool in Education

E-learning represents one of the most significant advancements in education, transitioning traditional teaching methods from rote memorization to creativity, interaction, and skill development (Hibi, 2024b). It encompasses all forms of electronic teaching and learning, leveraging the latest technologies in computing, storage, and networking to enhance educational delivery, publishing, and entertainment. This modern approach to education introduces dynamic opportunities for both educators and learners, making the learning process more accessible and engaging.

E-learning facilitates mastery of skills, simplifies study processes, and makes learning enjoyable (Albeshree et al., 2022). It plays a vital role in numerous governmental educational projects, including improving quality and accessibility, removing barriers to education, preparing students for employment, and enhancing workplace skills (Helsa & Juand, 2023). Despite its benefits, e-learning is not yet fully embedded at all educational levels, which highlights the need for greater integration and adaptation.

The value of e-learning lies in its ability to enhance educational quality while reducing costs. While not a replacement for teachers or lecturers, it acts as a complementary tool that builds an educated and adaptive workforce (Hibi, 2022). By employing interactive computers and direct internet connections, e-learning offers an unmatched level of flexibility, making it suitable for a wide array of learners and curricula across all educational stages.

The goals of e-learning are multifaceted. It aims to elevate educational standards, expand options for students, and assist underperforming learners in improving their academic achievements (Yohannes, & Chen, 2023). Additionally, e-learning helps reduce isolation by connecting students with peers and encouraging underprivileged communities to engage in learning.

E-learning is divided into two main types: synchronous and asynchronous learning. Synchronous learning utilizes global information networks to enable real-time interaction between students and educators through methods such as live chats and virtual classrooms (Juandi et al., 2021). One of the major advantages of this type of e-learning is the ability for students to receive immediate feedback. Asynchronous learning, on the other hand, allows learners to access pre-recorded sessions or planned lessons at their convenience. Tools like email and video recordings make this method highly adaptable, enabling students to revisit and review material whenever necessary.

The success of any educational program heavily depends on the educator's ability to convey information effectively. This has led to the emergence of educational software as an essential tool for supporting teachers and enhancing students' comprehension. Educational software, defined as multimedia-based instructional programs designed according to specific educational objectives, plays a crucial role in increasing the efficiency of learning processes (Zulnaidi et al., 2020). These programs, developed by experts, serve as a bridge between abstract content and interactive engagement, fostering better understanding and retention.

Educational software forms the backbone of e-learning. Its success represents a significant milestone in education, enabling institutions to fill gaps in traditional teaching methods. By facilitating both teaching and learning, these programs establish a knowledge-rich environment where learners can navigate freely and access diverse forms of information (Zhang et al., 2023). The advantages of educational software are numerous. It makes learning enjoyable and engaging, provides immediate feedback to learners, simplifies the presentation of material, and encourages active participation. It also optimizes the use of time, allowing students to learn at their own pace and level of ability. Moreover, it fosters collaboration among learners, creates a resource-rich environment, and standardizes the delivery of educational content (Radović et al., 2017).

Educational software stands out for its ability to present content in a learner-friendly manner, catering to individual preferences and needs. It organizes information scientifically and sequentially, taking into account prior knowledge and individual differences among learners. The interactive nature of such software ensures active learner engagement, offering drills, exercises, and instant feedback. By engaging multiple senses, these tools enhance knowledge retention and make the learning experience more effective and enjoyable.

In summary, e-learning and its associated technologies have redefined education, enabling more accessible, interactive, and engaging learning experiences. Through continued innovation and adaptation, these tools can bridge educational gaps, foster equity, and prepare learners for the demands of a rapidly evolving world.

(GSP): Revolutionizing Mathematics and Geometry Education

(GSP) has emerged as a transformative tool in mathematics and geometry education, offering students and educators an interactive platform that bridges the gap between abstract concepts and tangible learning. As a dynamic geometry software, it allows users to construct, manipulate, and explore geometric figures while simultaneously connecting these figures to algebraic relationships and mathematical theories. Its versatility has made it an invaluable resource across various educational levels, from elementary geometry to advanced calculus.

One of the most significant contributions of (GSP) is its ability to enhance students' conceptual understanding by providing visual and interactive representations of mathematical ideas. Traditional teaching methods often rely heavily on rote memorization, which can alienate students who struggle with abstract reasoning. (GSP), however, engages students through hands-on exploration, enabling them to experiment with geometric constructions and uncover relationships independently. Research has shown that such interactive learning environments significantly improve comprehension and retention of mathematical concepts (Cho et al., 2024).

The software's functionality goes beyond geometry, incorporating algebra, calculus, and trigonometry in a cohesive manner. It enables students to explore the relationship between equations and their graphical representations through visualization. For instance, users can plot functions and see immediate updates as variables are adjusted, promoting a stronger grasp of mathematical concepts. By addressing various learning styles—including visual, logical, and kinesthetic—it serves as an inclusive educational tool that supports improved learning outcomes for a wide spectrum of students (Cheng & Yeo, 2016).

(GSP) also encourages active learning by promoting critical thinking and problem-solving skills. Its interactive environment motivates students to hypothesize, test, and refine their understanding of mathematical principles. This process of discovery not only deepens their comprehension but also builds confidence in tackling complex problems. Furthermore, its ability to adapt to individual learning paces allows students to revisit and review material as needed, fostering autonomy and reducing anxiety associated with mathematics (Jones, 2012).

The software's benefits extend to educators, providing them with a dynamic tool to create engaging lesson plans. Teachers can use (GSP) to demonstrate complex ideas visually, simplifying explanations and increasing student engagement. Studies have highlighted that educators who incorporate the software into their teaching report higher levels of confidence and effectiveness, particularly in explaining intricate mathematical concepts (Öndeş, 2021).

While (GSP) has been widely adopted in educational systems worldwide, challenges remain in its implementation. Technical expertise among educators and budget constraints for acquiring software licenses can hinder its broader use. To address these challenges, professional development programs for teachers and institutional support for technological integration are essential (Emrouznejad, et al. 2023). Moreover, the availability of the software in multiple languages, including Arabic, and its compatibility with various devices make it accessible to a diverse audience, including remote and hybrid learning environments (Ferreira & Lima, 2020).

In conclusion, (GSP) has revolutionized mathematics and geometry education by transforming abstract concepts into engaging, interactive experiences. Its ability to integrate seamlessly across mathematical disciplines, coupled with its inclusivity for diverse learning styles, makes it a cornerstone of modern educational technology. As the role of technology in education continues to grow, (GSP) stands as a model for how innovation can enhance both teaching and learning.

PREVIOUS STUDIES

Several studies have examined the role of (GSP) in mathematics education, highlighting its effectiveness in enhancing learning outcomes, fostering engagement, and addressing specific learning challenges. These studies are based on diverse educational contexts and explore its impact on both students and teachers.

(Hollebrands & Lee, 2016) conducted a study that aligns with the growing body of research on the role of technology, such as the Geometer's Sketchpad (GSP), in mathematics education. Their work specifically examined how pre-service mathematics teachers implemented technology-based tasks with advanced middle-school students, highlighting how interactions between students, teachers, and technology evolved in the classroom. The study identified various ways teachers used technology to focus on mathematical concepts, either independently or in combination with technological features, and proposed a classification system for these interactions. This research underscores the potential of GSP in fostering deeper mathematical understanding and engagement by integrating technology effectively into teaching practices.

(Abu, M. et al., 2020) conducted a study focusing on the use of GSP in middle school classrooms. The research explored its impact on students' academic performance in analytic geometry and their attitudes toward mathematics. Working with a sample of 60 students, the study found a significant improvement in their understanding of geometric concepts and a more positive perception of the subject. These findings emphasize the effectiveness of dynamic geometry software in bridging the gap between abstract reasoning and practical application.

(Cevikbas & Kaiser, 2021) evaluated the role of GSP in professional development for mathematics teachers. The study included 35 secondary school teachers who participated in a training program designed to enhance their skills in teaching geometric proofs and transformations. The results showed that GSP significantly increased teacher confidence, helping them present complex mathematical ideas in a structured and accessible manner.

(Cayton et al., 2017) explored the impact of GSP on sixth-grade students' development of geometric intuition and visual thinking. Conducted with a group of 45 students, the research demonstrated that the interactive features of GSP allowed learners to visualize and manipulate geometric concepts effectively. This hands-on approach led to marked improvements in their spatial reasoning and overall engagement with the subject.

(Belbase, 2017) examined the role of GSP in helping middle school students master geometric transformations. The study included 50 participants and focused on topics such as reflections, rotations, and translations. The findings revealed that students using GSP achieved a deeper conceptual understanding and were better able to apply their knowledge in problem-solving tasks. The software's ability to facilitate experimentation and exploration was particularly beneficial in achieving these outcomes.

A study conducted by (Gyedu et al., 2017) addressed the challenges faced by students struggling with geometry. Working with 40 secondary school students identified as having difficulties in understanding geometric concepts, the research highlighted how GSP reduced student anxiety and made abstract ideas more tangible. This approach not only improved their performance but also fostered greater enthusiasm for learning mathematics.

This research uniquely explores the use of (GSP) in teaching parallelograms to fourth-grade students within the Arab sector in Israel, offering a culturally and contextually relevant perspective. By focusing on primary education, it addresses a critical gap in the literature, emphasizing the potential of dynamic geometry software to enhance foundational learning. What sets this study apart is its reliance on the insights of 100 fourth-grade teachers, highlighting the program’s effectiveness in improving student performance in geometry. The analysis also revealed that the program's impact was consistent across varying levels of teacher qualifications and years of experience, demonstrating its universal applicability. The study's methodological rigor, achieved through the use of a descriptive-analytical approach and SPSS for data analysis, reinforces the reliability of its findings. Practical recommendations to promote the broader adoption of dynamic geometry software (GSP) and improve teacher training highlight the research's value in advancing mathematics education. By tackling specific local challenges while providing universally relevant insights, this study makes a meaningful contribution to bridging the gap in integrating technology into early mathematics education.

STUDY METHODOLOGY

The researcher adopted the descriptive methodology, as it aligns with the purposes of this study. This methodology focuses on observing phenomena as they exist in reality, describing, analyzing, and linking them to other phenomena. The researcher relied on sources of information relevant to the study's topic, analyzed these sources, and subsequently collected data through a questionnaire. The questionnaire was developed based on the theoretical framework and previous studies. Study Population and Sample The study population consisted of teachers of fourth-grade students, selected randomly from various regions within the Arab community in Israel. A random sample of 100 teachers was chosen, and the demographic characteristics of the study sample are described as follows:

Table 1: Distribution of the Study Sample by Independent Variables.

Variable	**Classification**	**Frequency**	**Percentage (%)**
Gender	Male	39	39.0
	Female	61	61.0
Academic Degree	Bachelor's Degree	40	40.0
	Master's Degree	56	56.0
	Doctorate Degree	4	4.0
Years of Experience	Less than 5 years	53	53.0
	Between 5 and 10 years	38	38.0
	More than 10 years	9	9.0
Total		100	100.0

The table above illustrates the demographic distribution of the study sample, showing the frequency and percentage for each variable. The researcher employed a questionnaire as the primary study instrument after an extensive review of related literature and previous studies on the subject. The final version of the instrument consisted of 20 items. To ensure the validity of the instrument, it was presented to a panel of experts with specialization and experience in educational sciences and related fields. The experts provided feedback on the questionnaire items, offering suggestions for modifications, deletions, or the inclusion of additional items to ensure the instrument's relevance to the study's objectives. Reliability was assessed using Cronbach's Alpha coefficient to evaluate the internal consistency of the questionnaire items, resulting in a reliability coefficient of 90.7%, which indicated a high degree of reliability suitable for the study's purposes. The study was conducted following several key steps: first, the development of the final version of the study instrument; second, the identification of the study sample participants; third, the distribution of the questionnaires electronically to the sample; and finally, the entry and statistical analysis of the collected data using the Statistical Package for Social Sciences (SPSS). Following data collection, coding, and processing, appropriate statistical methods were applied using SPSS software, including frequencies, means, standard deviations, percentages, One-Way ANOVA, and Cronbach's Alpha. This meticulous process ensured that the findings were both reliable and valid, adhering to the highest standards of academic research.

RESULTS

Results Related to the Study Questions

This study aimed to examine the impact of using the (GSP) on teaching the topic of parallelograms to fourth-grade students and its effect on their academic performance in geometry, as perceived by teachers. To achieve this, the researcher utilized a 20-item questionnaire distributed to a sample of 20 teachers.

The questionnaire was designed based on a five-point Likert scale, with the items weighted as follows:

Table (2): Scoring Based on the Five-Point Likert Scale.

Response	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Score	5	4	3	2	1

The mean scores were converted into percentages and interpreted as follows:

- Above 80%: Very High
- 70% to less than 80%: High
- 60% to less than 70%: Medium
- 50% to less than 60%: Low
- Below 50%: Very Low

Results of the Main Study Question

The main question asked: What is the effect of using the (GSP) on teaching the topic of parallelograms to fourth-grade students and its impact on their academic performance in geometry, as perceived by teachers?

To answer this question, the means, standard deviations, and percentages for each domain of the tool were calculated. The following table presents the results:

Table (3): Means, Standard Deviations, and Percentages for the Impact of (GSP).

No.	Item No.	Domain	Mean	SD	Percentage	Rating
1	1	Geometry (General)	4.58	0.33	91.6%	Very High
2	2	Parallelograms (Specific)	4.40	0.39	88.0%	Very High
Overall Score			4.49	0.30	89.8%	Very High

The results in Table (3) indicate the following: The effect of using the (GSP) on teaching the topic of parallelograms to fourth-grade students and its impact on their academic performance in geometry, as perceived by teachers, was very high. Percentages ranged from 88.0% to 91.0% for the domains of Parallelograms (Specific) and Geometry (General). The overall percentage score was 89.8%, reflecting a significant positive impact.

Results of Sub-Question 1

The first sub-question asked: What is the impact of the (GSP) on fourth-grade students' academic performance in geometry? The following table summarizes the results:

Table 4: Means, Standard Deviations, and Percentages for the First Domain (Geometry - General).

No.	Item	Mean	SD	Percentage	Rating
1	I used (GSP) in teaching geometry topics	4.76	0.42	95.2%	Very High
2	I noticed improvements in students' understanding of geometry concepts	4.67	0.49	93.4%	Very High
3	The program facilitated geometric calculations and analyses	4.53	0.52	90.6%	Very High
4	The program motivated me to engage more in teaching geometry	4.37	0.59	87.4%	Very High
5	The program reduced the difficulty of teaching certain geometry concepts	4.54	0.57	90.8%	Very High
6	I observed academic improvement in geometry topics	4.59	0.55	91.8%	Very High
7	The program enhanced interaction and participation in geometry lessons	4.55	0.53	91.0%	Very High
8	The program positively impacted students' academic performance	4.55	0.60	91.0%	Very High
9	The program developed students' problem-solving skills in geometry	4.58	0.57	91.6%	Very High
10	Effectiveness of using the program in improving teaching experience	4.73	0.50	94.6%	Very High
Overall Score		4.58	0.33	91.6%	Very High

The results indicate that all items in the first domain (Geometry - General) were rated as very high, with percentages ranging between 87.4% and 95.2%. The highest-rated items were:

- "I used (GSP) in teaching geometry topics" (95.2%).
- "Effectiveness of using the program in improving teaching experience" (94.6%).

The overall score of 91.6% underscores the significant positive impact of (GSP) on students' performance in geometry.

Results of Sub-Question 2

The second sub-question asked: What is the impact of the (GSP) on fourth-grade students’ understanding of parallelograms?

Table 5: Means, Standard Deviations, and Percentages for the Second Domain (Parallelograms - Specific).

No.	Item	Mean	SD	Percentage	Rating
1	I used (GSP) to teach parallelogram concepts	4.25	0.60	85.0%	Very High

2	Improvements in students' understanding of relationships in parallelograms	4.17	0.60	83.4%	Very High
3	The program provided additional visual representations of parallelograms	4.42	0.63	88.4%	Very High
4	Simplified calculations and analyses of parallelograms	4.32	0.61	86.4%	Very High
5	Encouraged interest and participation in studying parallelograms	4.41	0.58	88.2%	Very High
6	Students benefited from practical exercises in parallelogram topics	4.46	0.57	89.2%	Very High
7	Enhanced students' readiness for geometric problem-solving	4.41	0.57	88.2%	Very High
8	Made learning parallelograms more engaging and effective	4.41	0.55	88.2%	Very High
9	Encouraged use of interactive tools in geometry teaching	4.40	0.53	88.0%	Very High
10	Recommended for improving teaching of parallelograms	4.70	0.502	94.0%	Very High
Overall Score		4.40	0.39	88.0%	Very High

All items in the second domain (Parallelograms - Specific) were rated as very high, with percentages ranging between 83.4% and 94.0%. The highest-rated item was: "Recommended for improving teaching of parallelograms" (94.0%). The overall score of 88.0% highlights the substantial impact of (GSP) in enhancing students' understanding of parallelograms.

Results Related to the Study Hypotheses

First Hypothesis

The first hypothesis states: There are no statistically significant differences at the level ($\alpha = 0.05$) in the effect of using the (GSP) software on the performance of fourth-grade students in geometry from the teachers' perspective, based on differences in academic qualifications.

To examine the validity of this hypothesis, a one-way ANOVA was used. The results are detailed in the following tables:

Table 6: Means and Standard Deviations for Academic Qualification Variable (Geometry).

Academic Qualification	N	Mean	Std. Deviation
Bachelor's Degree	40	4.57	0.34
Master's Degree	56	4.60	0.33
Doctorate	4	4.55	0.41
Total	100	4.58	0.33

The table shows differences in the mean scores for the academic qualification variable in geometry. To test the significance of these differences, a one-way ANOVA was performed, as shown in Table 7.

Table 7: One-Way ANOVA Results for Academic Qualification (Geometry).

Source of Variance	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.029	2	0.015	0.126	0.882
Within Groups	11.264	97	0.116		
Total	11.293	99			

The results indicate no statistically significant differences at ($\alpha = 0.05$) between the mean scores of the study sample concerning the effect of using (GSP) on the performance of fourth-grade students in geometry based on academic qualifications. The α -value (0.882) is greater than (0.05), leading to the acceptance of the null hypothesis for this variable.

Second Hypothesis

The second hypothesis states: There are no statistically significant differences at the level ($\alpha = 0.05$) in the effect of using the (GSP) software on the performance of fourth-grade students in geometry from the teachers' perspective, based on differences in years of experience. A one-way ANOVA was also used to examine this hypothesis, and the results are presented below:

Table 8: Means and Standard Deviations for Years of Experience Variable (Geometry).

Years of Experience	N	Mean	Std.
Less than 5 years	53	4.55	0.33
5–10 years	38	4.66	0.29
More than 10 years	9	4.46	0.50
Total	100	4.58	0.33

The results indicate differences in the mean scores across the years of experience variable. To test their significance, a one-way ANOVA was performed, as shown in Table 9.

Table 9: One-Way ANOVA Results for Years of Experience (Geometry).

Source of Variance	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.391	2	0.195	1.739	0.181
Within Groups	10.902	97	0.112		
Total	11.293	99			

The data indicate no statistically significant differences at (α = 0.05) between the mean scores concerning the effect of (GSP) on performance in geometry based on years of experience. The α -value (0.181) is greater than (0.05), confirming the acceptance of the null hypothesis for this variable.

Third Hypothesis

The third hypothesis states: There are no statistically significant differences at the level (α = 0.05) in the effect of using (GSP) on the performance of fourth-grade students in parallelograms from the teachers' perspective, based on differences in academic qualifications.

A one-way ANOVA was performed, and the results are summarized in the following tables:

Table 10: Means and Standard Deviations for Academic Qualification Variable (Parallelograms).

Academic Qualification	N	Mean	Std.
Bachelor's Degree	39	4.38	0.30
Master's Degree	56	4.40	0.44
Doctorate	4	4.57	0.37
Total	100	4.40	0.39

Table 11: One-Way ANOVA Results for Academic Qualification (Parallelograms).

Source of Variance	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.136	2	0.068	0.441	0.645
Within Groups	14.773	96	0.154		
Total	14.909	98			

The p-value (0.645) exceeds (0.05), indicating no statistically significant differences between the mean scores regarding the effect of (GSP) on performance in parallelograms based on academic qualifications.

Fourth Hypothesis

The fourth hypothesis states: There are no statistically significant differences at the level (α = 0.05) in the effect of using (GSP) on the performance of fourth-grade students in parallelograms from the teachers' perspective, based on differences in years of experience.

Table 12: Means and Standard Deviations for Years of Experience Variable (Parallelograms).

Years of Experience	N	Mean	Std.
Less than 5 years	52	4.38	0.32
5–10 years	38	4.46	0.33
More than 10 years	9	4.25	0.80
Total	100	4.40	0.39

Table 13: One-Way ANOVA Results for Years of Experience (Parallelograms).

Source of Variance	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0.384	2	0.192	1.269	0.286
Within Groups	14.525	96	0.151		
Total	14.909	98			

The results confirm no statistically significant differences at ($\alpha = 0.05$) based on years of experience, as the p-value (0.286) is greater than (0.05). The null hypothesis is therefore accepted.

DISCUSSION

The results of this study confirm that the use of (GSP) significantly enhances students' understanding of parallelograms and their academic performance in geometry. This aligns with prior studies that have demonstrated the efficacy of interactive software in fostering conceptual understanding and engagement in mathematical learning environments (Eshetu et al.,2022).

The absence of statistically significant differences across variables such as teachers' qualifications and years of experience underscores the universal applicability of this software. Regardless of professional background, teachers found (GSP) to be an effective teaching tool. These findings resonate with those of (Kartal & Çınar 2024), who highlighted the accessibility and adaptability of dynamic geometry software in diverse educational contexts.

Teachers reported a significant improvement in students' comprehension of parallelograms, supported by visual and interactive learning environments. This finding echoes (Gyedu et al., 2017) research, which emphasized the role of interactive software in reducing students' anxiety and improving engagement with complex geometric concepts.

The study revealed that (GSP) not only benefited students but also empowered teachers by simplifying the delivery of abstract concepts. Similar results were found in (Hollebrands & Lee, 2016) study, which demonstrated increased teacher confidence when integrating dynamic software into lesson plans.

The results revealed no significant variation in effectiveness based on teachers' qualifications or years of experience. This highlights the intuitive design and user-friendliness of (GSP), aligning with findings by (Cheng & Yeo, 2016), who noted the minimal training required for teachers to effectively use such tools.

The study contributes to the growing body of evidence supporting technology integration in early mathematics education. For the Arab sector in Israel, this research highlights the potential of tools like (GSP) to address systemic educational challenges, enhance teacher pedagogy, and improve student outcomes in geometry.

RECOMMENDATIONS

Integrating (GSP) into the curriculum is essential to standardize its use across classrooms and ensure equal learning opportunities for all students. Comprehensive teacher training is required to enable educators to fully utilize the software's capabilities. These training programs could include dedicated workshops, practical exercises, and ongoing support for teachers.

To address existing achievement gaps, particularly in the Arab sector, initiatives should promote the use of dynamic geometry tools to foster equitable learning opportunities, especially for students who are underperforming. Additionally, long-term research should investigate the impact of (GSP) on mathematical reasoning, its effects on other branches of mathematics, and conduct comparative studies across schools with diverse socio-economic backgrounds to identify effective practices.

Improving the accessibility of the software in languages such as Arabic and Hebrew is crucial to meet the linguistic needs of diverse populations. Moreover, integrating (GSP) into modern teaching platforms can facilitate the adoption of innovative hybrid learning models. Using the software to design interactive lesson plans and applied assignments can help students deeply understand complex mathematical concepts. Furthermore, its application can be extended to scientific fields such as physics and engineering, offering visual demonstrations of processes and phenomena.

Enhancing the user experience through a user-friendly interface and clear tutorials will help new users quickly grasp the software's core functionalities. Additionally, incorporating the software into assessments, including tests and exams, can encourage creative use of dynamic tools alongside traditional computation methods.

Creating an active community of users, including teachers and students, will enable the sharing of knowledge and ideas for practical application of the software. Digital equity can be promoted by providing subsidized licenses to schools in underprivileged areas.

Integrating the software with emerging technologies, such as 3D printers, can allow students to translate theoretical concepts into tangible physical models. Advanced analytical capabilities within the software, offering personalized feedback, can enhance learning and enrich the user experience.

Finally, collaborations with academic institutions can foster research to examine the software's impact on educational outcomes. Additionally, tailored content for advanced students can be developed to encourage creative thinking and the cultivation of advanced mathematical skills.

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